Resumos

MS3: IntMath-TSD: Interval Mathematics and Connections in Teaching and Scientic Development

- **Título: Improving the Efficiency of Dot Product Computations Using Error Free Transformations in C-XSC**

  Prof. Dr. Gerd Bohlender (KIT, Karlsruhe)

Abstract: Dot products are one of the most frequent operations in numerical computations. A special feature of the C++-class library C-XSC for scientific computing is the computation of such dot products with high accuracy. A long accumulator is provided in the form of special dot precision variables which may store the intermediate result of a dot product without any rounding error. Only the final result has to be rounded to double precision. This method avoids cancellation errors and thus improves the accuracy and reliability of numerical algorithms. It also makes the error analysis of numerical methods much easier. Using adequate hardware support, dot product computations in a long accumulator could be made as fast as conventional floating-point approximations. Many algorithms and hardware designs have been developed for this purpose. Unfortunately, in the currently widespread hardware architectures there is no such hardware support. Therefore, the long accumulator has to be simulated in software which makes it considerably slower than floating-point approximations. In recent versions of C-XSC, dot products with selectable precision have been introduced. These implementations are based on error free transformations and the so-called DotK algorithm. We briefly explain the theoretical background of these algorithms and show how the user can select a compromise between speed and accuracy. Many variants are possible from conventional floating-point (least accurate but fastest) via several intermediate variants to exact dot products using the long accumulator (most accurate but slowest, when simulated in software). We present many numerical examples using these variants of dot products: simple arithmetic operations, numerical algorithms from the text book, and numerical algorithms with verified results using interval arithmetic. In all examples we discuss the accuracy of the results and the efficiency of the computation.

- **Título: Verified Solver with High Accuracy for Resolution of Dense Linear Systems**

  Prof. Dr. Carlos Holbig (UPF/RS)

Abstract: This paper presents a study of parallel versions of the verified Linear System Solver (LSS), and based on this study presents a new parallel version of LSS based on optimizations performed in its original sequential version. These optimizations in sequential version and the consequent development of a new parallel version aimed at an improvement in computational performance of the solver LSS without losing the characteristic of high accuracy. The solver

LSS, in its various versions, is used for solving dense linear systems of the type $Ax = b$, treating numerical instability of arithmetic floating-point machine through the C-XSC library with the use of interval mathematics. In addition, we will present the development history of this solver, since its first implementation in C-XSC in 2002 until the present day.

- **Título: Intervals on Self-verified Linear Systems Solvers on Multicore Computers**

  Prof. Dr. Luiz Gustavo Fernandes (PUCRS/RS)

Abstract: Several works have focused on optimizing Verified Computing performance for computer clusters. However, many changes have been occurring in high performance computing. Given the number of cores on multicore chips expected to reach tens in a few years, efficient implementations of numerical solutions using shared memory programming models is of urgent interest. In this context, we developed a self-verified solver for dense interval linear systems optimized for parallel execution on multicore processors. Two techniques were employed on the development of the solver. The first one was to optimize the matrix inversion step of algorithm by employing PLASMA routines. The second was to divide the computation of the interval iteration matrix bounds by using different threads to execute the operations in each rounding mode. The adopted strategies have resulted in a scalable solver that obtained up to 85% of reduction at execution time and a speedup of 6,70 with efficiency nearly to 84% when solving a 15.000 X 15.000 interval linear system on an eight cores computer.

- **Título: Parallel Self-verified Linear System Solver on Cluster Computers**

  Prof. Dr. Mariana Luderitz Kolberg (PUCRS/RS)

Abstract: This research presents a free, fast, reliable and accurate solver for point and interval dense linear systems. The idea was to implement a solver for dense linear systems on cluster computers using a verified method, interval arithmetic and directed roundings based on MPI communication primitives associated to optimized libraries, aiming to provide both self-verification and speed-up at the same time. We proposed and implemented a new sequential verified solver for dense linear systems for point and interval input data using both infimum-supremum and midpoint-radius arithmetic based on highly optimized libraries (BLAS/ LAPACK). Performance tests showed that the midpointradius algorithm needs approximately the same time to solve a linear system with point or interval input data, while the infimum-supremum algorithm needs much more time for interval data. Considering that, midpoint-radius arithmetic was the natural choice for the next step of this work: the parallel implementation.

- **Título: Using Field Programmable Gate Array (FPGA) to implement hardware accelerator for scientific computing applications**

  Prof. Dr. Abner Corrêa Barros (UFPE/BRAZIL)

Abstract: Logic devices such as Field Programmable Gate Arrays (FPGAs) are increasingly being used in real-world high performance scientific computing problems as hardware accelerators, due to the following reasons: *(i)* the intrinsic parallelism associated to their internal architectures; *(ii)* fast functional customization; and *(iii)* good computational density per watt, associated to the possibility to implement customized arithmetical operators. This work presents a methodology for the development and analysis of arithmetic accelerators using FPGAs devices applied to scientific computing applications. The proposed model is based on an incremental methodology approach, starting with a canonical model and it develops to a high level synthesizable RTL model. This methodology was successfully applied in two case studies: in the design of a double precision floating-point multiplier with accumulator (MAC), and a finite difference operator for seismic applications operating on single precision floating-point numbers.

- **Título: Analysis of Transient Phenomena in Turbulent Flows using Wavelets**

  Prof. Dr. Maria Luiza Indrusiak (UNISINOS/RS)

Abstract: Engineering phenomena are essentially nonstationary, despite the usual approach that considers them as quasi-stationary or as a succession of stationary ones. Nevertheless, the analysis of transient features in engineering processes is important in order to enhance the understanding about them. The improvement of the process control and efficiency are the main goals. Wavelet analysis is the best tool to apply in such cases. The objective of the seminar is to present some applications of the continuous and discrete wavelet analysis and results obtained. A bistable flow inside a tube bank, a transient vortex street and a boundary layer detachment were analyzed with new and interesting results. Some new ideas for research in transient phenomena using wavelets are also presented.

- **Título: Interval Mathematics in Scilab**

  Prof. Dr. Maria Angélica Camargo Brunetto (UEL/PR)

Abstract: The main objective is to present some functionalities of the Int4Sci Toolbox, a Scilab Interface for Interval Analysis. Extended interval arithmetic, elementary functions, interval evaluation, interval characteristics calculation are included in Scilab, as functions applied to intervals and interval manipulation.

- **Título: Interval Mathematics: A Real Application to Electric Power Systems**

  Prof. Dr. Luciano Vitória Barboza (IFSUL/Pelotas)

Abstract: This lecture focuses on the application of Interval Mathematics techniques to the power flow tool in electric energy systems. It is well known that the load flows studies are the most important one in both supervisory and control centers of electric systems. On the other hand, it is known that the major shortcoming of this tool is the use of punctual data, that is, power demand levels in a specific time. This way, for the mathematical modeling of the electric network equations, we use a steady state representation of the system in which the demand variations occur in a slow way so that the electric system tends to converge a new operational point. Nevertheless, we know that any demand variations in the real and reactive power loads imply in change on both the nodal voltage profile and power flows in the circuits. Using real and reactive power demands as intervals it is possible to perform a study of the electric network behavior under power demand variations. Thus, we can also obtain the nodal voltage profile and the power flows as interval data. This new approach to deal with the basic data for power flow studies was named "timing load flow".

- **Título: GMC - Computing Mathematics Group**

  Prof. Dr. Liara Aparecida dos Santos Leal (PUCRS/RS)

Abstract: This article has the purpose to present a bit of the history of the group search GMC-Computing Mathematics Group, headed by Prof. Dr. Dalcidio Moraes Claudio and those formation, originally, was due to the initiative of a group of teachers from the Institute of Informatics of the Federal University of Rio Grande do Sul (UFRGS) in the early 1980s. Over nearly 30 years of expertise in the scenario of scientific research in Brazil and abroad, the Group GMC currently headquartered in Pontifícia Universidade Católica do Rio Grande do Sul (PUCRS), has developed research activities, mainly in areas related to Computational Mathematics and Informatics in Education. The development of research projects covering the areas of Mathematics, Computer Science and Education, brought a multi-disciplinary group, whose diversity of ideas, people and interests, resulted in a heterogeneous group and collaborative, with motivation for intellectual and spiritual growth. The history of GMC has as protagonists each of students, scholars of, technicians, MA students, PhD students, teachers, mentors, people who created its own history along their own way, but certainly also contributed to the history of scientific development in our country.

- **Título: An Interval Probability based on Interval Analysis and High Accuracy Arithmetic**

  Prof. Dr. Marcília Andrade Campos (UFPE/PE)

Abstract: We propose an interval-valued probability, and we prove that this probability owns several properties. This probability is denoted by an interval probability or a self-validated probability. We also show as interval-valued discrete and continuous random variables can be defined. IntLab is used for computing numerical examples.

- **Título: Q-calculus**

Prof. Dr. Benedito Melo Acioly (ESB/BH)

Abstract: The quantum calculus or q-calculus begins with F. H. Jackson in the early twenties, but only recently it has aroused interest, due to great demand for mathematics to model quantum computations. The q-calculus considers the following expression $\frac{f(x)-f(x_0)}{x-x_0}$, derived from the usual differential calculus, and instead of taking the limit when x tends to x_0 in order to obtain the concept of derivation at the point x_0,, it considers $x = q\ x_0$ or $x = x_0 + h$. In addition, it does not consider the limit but fixes q or h. With this change we enter in the fascinating world of quantum calculus.

- **Título: Some Continuity Notions for Interval Functions and Interval Representations**

Prof. Dr. Benjamin Callejas Bedregal (UFRN/RS)

Abstract: The notion of continuity plays an important role in many fields of human activity. Ramon Moore introduced a distance metric for the set of real intervals and so a continuity notion for interval functions. He also showed that every (rational) continuous real function can be extended to a continuous interval function. Moreover, Dana Scott demonstrated that IR fitted with reverse order of inclusion is a non-algebraic continuous domain. This structure has two equivalent notions of continuity: *(i)* a topological, and *(ii)* an order point of view. Recently, Acióly and Bedregal defined a quasi-metric for *IR* and proved that the topology induced by this such quasi-metric is exactly the Scott topology for *IR*. In addition, the metric underlying the quasi-metric is the Moore metric. Futhermore, Santiago, Bedregal and Acióly showed some relationships between continuous functions of these topologies. This work considers two new notions of continuity which are consequences of three different interval semantic views: interval as a generalization of a number, as information and as a set. These views lead to the topology of Moore, the Scott topology and the topology induced by the dual Scott topology, respectively. The set of viewpoints and information lead to the bicontinuity notion which is explored here. It is also seen how the order of Kulisch and Miranker can induce a quasi-metric. The comparison between the notion of continuity induced by this quasi-metric and the other notions of continuity is analysed.

- **Título: Informational Distances: Introducing Interval Distances**

Prof. Dr. Regivan Hugo Nunes Santiago (UFRN/RN)

Abstract: One of the first notions of distance is that of metric, which captures the intuitive properties with respect to a notion of distance between two objects. However, such formalization is not universal, some other very important notions of distances have been proposed during the time, some of them are more abstract and aim to provide more information during the process of measurement (i.e. quasi-metrics, hyper-metrics, semi-metrics). All of them are obtained by changing the axiomatics of metrics; however all of them

are functions of the same form. In this talk, we propose a generalization of metrics with respect to the generalization of their value space, we propose a more general structure. The resulting function are more informative functions (that's why the name Informational Distances) which give the value of a distance together, with a possibly, extra-information. One special case, is that of Interval Distances, where the result of a measurement is an interval which formalizes the statement: The distance between *a* and *b* is *d* with an error of "ε".

- **Título: Interval Mathematics and Signal Processing**

  Prof. Dr. Roque Mendes Prado Trindade (ESB/BH)

Abstract: Nowadays, there is no denying the importance of signal processing in engineering, as well as the presence of noise and uncertainties in signal processing. Although most signals are continuous, processing is predominantly discrete, so this work considers theoretical foundations of interval mathematics as an appropriate approach to deal with signal processing, since it can represent the continuous signals in the discreet ones.

- **Título: Interval Valued Fuzzy Logic**

  Prof. Dr. Renata Hax Sander Reiser (UFPEL/RS)

Abstract: Interval valued fuzzy logic emerged in 1975 with independent contributions from various research groups, aiming at the treatment of uncertainty not only about the membership functions but also about the membership degree of relevance, which means, the notion of "precise number" representing a membership degree is extended to a value carrying its uncertainty in the unity interval. In this sense, the extensions of the interval fuzzy connectives have been widely studied. In interval mathematics, the result of an interval computation must always contain the value of the related real function ensuring the correctness of its computations. In this context, interval representations are able to express computations with real numbers and their interrelations in Scott and Moore topologies. Based on the canonical interval representation of a real function, also aggregating the optimality criteria, it is possible to obtain the correct and optimal (lowest) result, but not necessarily a computable one. Our task has undertaken the definition of canonical interval representation within fuzzy connectives, emphasizing dual constructions as the interval valued fuzzy coimplications. The canonical interval constructor preserves extensions of continuous and monotonic inclusions of such fuzzy connectives, including similar properties of related dual interval valued fuzzy implications.