Decay and control for a double membrane system

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SUMMARY

Our goal is to study the behavior of the energy and controllability from the boundary for the hyperbolic system

$$u'' - \Delta u + a_1 u + b_1 v = 0,$$
$$v'' - \Delta v + a_2 v + b_2 u = 0.$$

A physical problem modeled by this type of system may be found in [4]. The functions u and v describe the transverse vibrations of an elastically conected double-membrane system. As in [1] and [3], we assume that the equations have the following form

$$u'' - \Delta u + k(u - v) = 0,$$
$$v'' - \Delta v + k(v - u) = 0,$$

where k is a positive constant.

The Cauchy problem for such system of equations is the following

$$u'' - \Delta u + k(u - v) = 0 \ in \ R^2 \times R,$$
$$v'' - \Delta v + k(v - u) = 0 \ in \ R^2 \times R,$$
$$u(0) = u_0, \quad u'(0) = u_1 \ \ in \ \ R^2,$$
$$v(0) = v_0, \quad v'(0) = v_1 \ \ in \ \ R^2.$$

The energy of the solution of the Cauchy problem confined in a bounded domain $U \subset R^2$ at the time $t > 0$ is given by

$$E(U, t) = \frac{1}{2}\int_U \{|u'|^2 + |v'|^2 + |\nabla u|^2 +$$
$$+ |\nabla v|^2 + k|u - v|^2\} dx$$

In this work we prove that for each U there exists $M = M(U,k) > 0$ such that

$$E(U,t) \leq \frac{M}{t^2} E(U,0)$$

for every $t > diam(U)$.

As in [2], we use this decay estimate to obtain exact boundary controllability for the finite energy solutions of the system. More precisely we obtain the following result:

Theorem: Let $\Omega \subset R^2$ be a curved polygon. There exists $T > 0$ such that for any functions $u_0, v_0 \in H^1(\Omega)$, $u_1, v_1 \in L^2(\Omega)$ there exist $g_1, \ g_2 \in L^2(\partial\Omega \times [0,T])$ so that the solution of

$$u'' - \Delta u + k(u - v) = 0 \ in \ \Omega \times]0,T[,$$
$$v'' - \Delta v + k(v - u) = 0 \ in \ \Omega \times]0,T[,$$
$$u(0) = u_0, \quad u'(0) = u_1 \ \ in \ \ \Omega,$$
$$v(0) = v_0, \quad v'(0) = v_1 \ \ in \ \ \Omega,$$
$$\frac{\partial u}{\partial \eta} = g_1, \frac{\partial v}{\partial \eta} = g_2 \ in \ \partial\Omega \times]0,T[,$$

satisfy $u = u' = v = v' = 0$ in Ω at the time T. Here H^1 and $L^2 = H^0$ are the usual Sobolev Spaces. □

References

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