Spectral hp Elements for Flow in Porous Media

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RESUMO

Numerical simulation is a powerful tool deployed in the various branches of engineering and science for the development of models an analysis of real problems. In engineering, it is used during the design stage to analyze the performance, resistance, feasibility and safety of an equipment or process being devised. In science, its application aims basically at getting a better understanding of the phenomenon under study.

In petroleum engineering, and specifically in reservoir engineering, it is a common practice the use of numerical reservoir simulators for the study of alternative recovery procedures for the optimization of hydrocarbon extraction and the forecast of reservoir performance and the impact of operational decisions.

Many of the reservoir simulators used nowadays are based on methods of finite differences, which approximate the differentials of the equations by suitable finite differences of the functions. These methods have the disadvantage of needing a regular grid for the discretization process. As in any numerical method, there are also issues of convergence and stability that must be addressed. In general, to increase the order of accuracy of an approximate solution it is necessary to refine the spatial grid and reduce the time steps, increasing the computational effort.

There are also reservoir simulators based on the technique of finite element methods. The finite element methods were developed initially to solve structural mechanics problems, but they proved to be successful also in solving fluid dynamics problems. The finite element method has the advantage of being able to use unstructured domain decomposition, which makes it easy to deployed in the study of problem with irregular or complex spatial domain. However, the order of the local polynomial approximation is in general low. Quadratic convergence is usually sufficient for stationary problems, but for time-dependent problems, where long time integration is required and errors may be amplified, it is desirable to have high-order spatial approximations.

The spectral h/p elements have arisen from the combination of ideas of spatial discretization of finite element methods with the high-order local approximations of multi-domain spectral methods. It consists basically in partitioning the spatial domain in small regions, as done in Finite Element Methods (FEM), and in locally approximating the functions over these regions by an expansion in terms of suitable base of functions, usually Chebyshev polynomial, Legendre polynomials or general Jacobi polynomials.

Here we present an implementation of the spectral h/p element for the numerical solution of the problem of the flow of two immiscible fluids in a porous media. We find a weak solution using the Galerkin formulation of a 2-D problem using a triangular discretization with a local approximation of the functions in terms of Jacobi polynomials. The algorithm is implemented in a C++ code which makes it easy the incorporation of 3-D elements and other problems. We compare our results with the results of commercial simulators (CMG) which are based on finite differences.