Variational derivation of immersed domains methods for fluid–solid interaction problems

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ABSTRACT

In the present work a variational principle for the fluid–solid interaction problem employing the concepts of immersed domains is derived from a well–known governing principle from the continuum mechanics. The goal of this work is to present a general formulation in which an augmented fluid, composed by the real fluid and a fictitious fluid that overlaps with the solid domain, interacts with an immersed solid. The formulation was introduced and analyzed in detail in [1]. It provides a general framework for fluid–solid interaction problems with respect to the compressibility characteristics of both fluid and solid. Although it is very close from formulations like the immersed finite element method [2] and the immersed continuous method [3], one of the differences is that in the present approach the volume force in the fluid domain emerges as a reactive force due to the imposition of the continuity in the velocity field along the whole overlapped domain by means of a Lagrange multiplier. Another important difference lies in the manipulation of the mass conservation for the artificial fluid. Here we get rid of the restriction to compressible fluid–compressible solid of the immersed continuous method and the incompressible fluid–incompressible solid of the immersed finite element method.

In our approach, the Euler–Lagrange equations of the system are attained by relaxing the continuity of the fluid and solid velocity fields in the solid domain through a Lagrange multiplier. Then, it should be said that we propose a generalization of a fictitious domain method in which the solid domain is deformable and the fictitious fluid inherits its compressibility properties in order to conciliate the arbitrariness in the compressible character of both elements. Also, a numerical approximation of the problem restricted to fluid–rigid body interactions is carried out and several examples are presented.

Referências

[1] P.J. Blanco, "Kinematical incompatibility, immersed domains and multiscale constitutive modeling: nexus with the modeling of the cardiovascular system", (in portuguese) Ph.D. Thesis, LNCC/MCT, 2008.

[2] W.K. Liu, D.W. Kim and S. Tang, Mathematical foundations of the immersed finite element method, *Comput. Mech.*, 39 (2007) 211-222.

[3] X.S. Wang, From immersed boundary method to immersed continuum method, *Int. J. Mult. Comp. Engrg.*, 4 (2006) 127-145.