 ISSN 1984-820X

A Fuzzy-parameter Dispersion-attraction Partial Differential Model for an Ant Colony

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Abstract: *This paper proposes a model for the occupation of ants in a region of attraction, using evolutive partial differential diffusion-advection equations, in which the population dispersion and velocity in direction x and y are fuzzy parameters. The domain under study contains an attractive region, representing areas with high concentrations of palatable host-plants with better nutritious qualities. The algorithm developed here uses information about the foraging behavior of a leaf-cutting ant colony of the Amazon region in northern Brazil. We calculated the numerical solution of the partial differential equation, determining the dispersion at each iteration and for each triangular finite element. The dispersion was determined from a fuzzy rule-based system that depends on the number of individuals of the population and on the characteristics of the terrain. In this model, we also considered the velocity along the x and y directions as a fuzzy parameter that depends on the ant movement characteristics on the terrain in the y and x directions, respectively. The model produced solution consistent with ant behavior in response to the proximity of an attractive region, as stated in the literature [2], this model incorporates uncertainties that do occur in the biological phenomenon under study.*

Palavras-chave: *Partial differential equation, fuzzy set, numerical methods, leaf-cutting ants*

1 Introduction

Ants of the genera *Atta* and *Acromyrmex* (*Hymenoptera: Formicidae*), collectively known as leaf-cutting ants, have the unique habit of culturing fungus on fresh plant materials. These ants are generalist foragers that exploit a large number of plant species, although they usually focus on a subset of these species [5], especially those with low levels of toxic secondary compounds and high nutrient content. Their system of foraging trails influence the spatial organization of the foraging activity. Spatial and temporal heterogeneity in plant resources within the vicinity of the ants' nest produces changes in trail direction and modifications in the geometry of the foraging territories. Trails often lead to sites where, at a given time, plant resources are more attractive and abundant. For instance, a study in Costa Rica showed that, when leaf-cutting ant colonies have access to different plant communities, they concentrate their foraging efforts on the community with the greatest density of their preferred plant species (a monoculture of cassava) [2].

This paper proposes a model for the occupation of an attractive area by leaf-cutting ants, using a form of the Diffusion-Advection partial differential equation to model population dispersal and velocity using a fuzzy parameter. In this work, dispersion is determined through a fuzzy rule-based system that depends on the number of individuals of the population and the characteristics of the environment through which insects travel. The velocity along the x is also determined by means of a fuzzy rule-based system that depends on the characteristics of the terrain in the direction of y and, analogously, on the velocity along the y. The model is constructed using expert entomological knowledge information on behavior of leaf-cutting ants. The domain of this study contains an attractive region, which can represent areas with a high concentration of palatable host plants with good nutritious qualities. In literature, these researches usually involve the use of a deterministic or stochastic model. However, mathematical literature on uncertainty has grown considerably over the last decade, especially in system modeling, optimization, control, and pattern recognition areas, to mention just a few. Recently, several authors have proposed the use of fuzzy set theory in epidemiology problems [3], [4] and population dynamics. In this paper, we suggest fuzzy set theory, introduced in the 1960s by Lotfi Zadeh, to deal with the uncertain nature of ant population dispersal. The present study is based on the fact that the availability of plant resources to leaf-cutting ant colonies varies considerably in space and time. Thus, colonies must continually gather information about the current availability or stock of resources within their territories. Once a patch of palatable vegetation (i.e., an attractive area) is found, a chemically and physically marked trail is established and the ant foragers follow this trail in order to retrieve the newly found resource.

2 Ant Study Area in Amazonas

This study was carried out in a forest reserve located about 70 kilometers north of Manaus, state of Amazonas, Brazil. The vegetation is that of a primary inland rain forest, which, unlike other types of forest in the Amazon basin, is not subject to annual flooding. The climate is tropical and the annual precipitation, which is about 2,100 mm, is seasonal in distribution. The rainy season lasts from November to May, and the dry season from June to October.

Vasconcelos's research [6] involved a study of the foraging activity of an *Atta cephalotes* colony from July 1985 to January 1986 and from September 1986 to March 1987. This study determined the spatial distribution of ant foraging trails and of the plant species exploited by the ants (i.e., their foraging sites).

Figure 1 shows that the foraging sites of the *Atta cephalotes* were scattered over the entire extent of the trails, though mainly at intermediate distances from the nest (40 - 60m).

3 Fuzzy Model

The study of population dynamics involves several uncertain variables, such as the number of individuals in the population, the landscape, the speed at which the population travels, and population dispersal. Fuzzy set theory is a mathematical tool for modeling uncertain phenomena. Thus, the aim of this research is to find a numerical solution for modeling the presence of ant populations, treating dispersal and velocity as a fuzzy parameter [1].

The model proposed for the occupation by leaf-cutting ants will be studied by means of the diffusion-advection partial differential equation, given by:

$$\frac{\partial P}{\partial t} + v(loc) \cdot \nabla P - \nabla \cdot (\alpha(P, loc_{tot}) \nabla P) = 0. \tag{1}$$

The functional variable $P = P(\mathbf{x}, t)$ indicates the population at the instant $t \in [0, T]$ and at the point $\mathbf{x} \in \Omega_1 \subset \mathbf{R}^2$ (see Figure 2). Thus, we assume that population dispersal is represented by parameter $\alpha(P, loc_{tot})$, with total locomotion (loc_{tot}) determined by a fuzzy rule-based system that depends on the input variables loc_x and loc_y. Population velocity is $v(loc) = (v_1(loc_y), v_2(loc_x))$, where loc is a vectorial function, $loc = (loc_x, loc_y)$, which provides the values for locomotion along the x and y. This velocity depends on the subregions of the

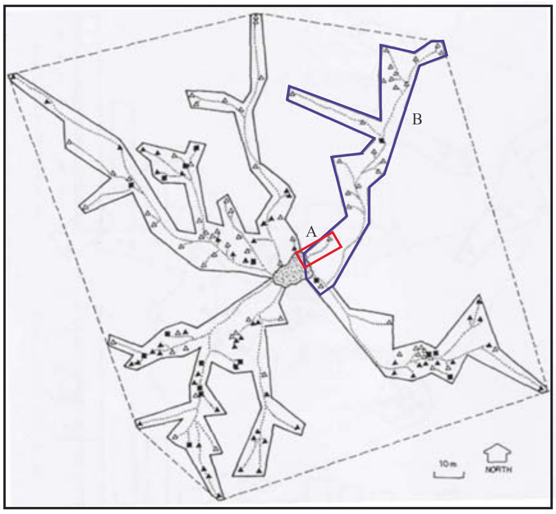

Figure 1: Foraging range (broken line) and foraging territory (continuous line) of the *Atta cephalotes* colony. Solid triangles represent the plants attacked between July 1985 and January 1986, and empty triangles those attacked between September 1986 and March 1987. Solid squares represent plants attacked during both periods. Dotted lines represent the foraging trails radiating from the nest (shaded area) [6].

domain, indicating the ants' difficulty of locomotion over the terrain. The fuzzy parameters are modeled as follows:

- the domain is divided into subregions;
- the characteristics of the environment in which the individuals travel are represented by trapezoidal membership functions that indicate the degree of difficulty in locomotion through the domain;

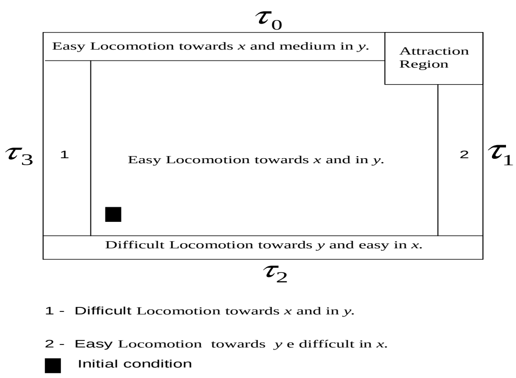

Figure 2: Domain Ω_1.

The adopted boundary conditions considered are:

$$\alpha(P, loc_{tot})\frac{\partial P}{\partial y} = Pv_2(loc_x), \qquad \forall \mathbf{x} \in \tau_0, \qquad \forall t \in [0,T] \tag{2}$$

$$\alpha(P, loc_{tot})\frac{\partial P}{\partial x} = Pv_1(loc_y), \qquad \forall \mathbf{x} \in \tau_1, \qquad \forall t \in [0,T] \tag{3}$$

$$\alpha(P, loc_{tot})\frac{\partial P}{\partial y} = Pv_2(loc_y), \qquad \forall \mathbf{x} \in \tau_2, \qquad \forall t \in [0,T] \tag{4}$$

$$\alpha(P, loc_{tot})\frac{\partial P}{\partial x} = Pv_1(loc_y), \qquad \forall \mathbf{x} \in \tau_3, \qquad \forall t \in [0,T]. \tag{5}$$

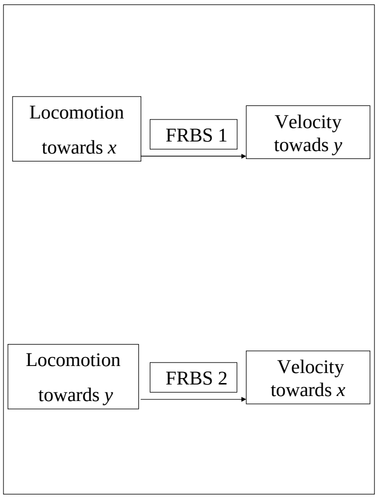

Figure 3: Fuzzy rule-based systems 1 and 2.

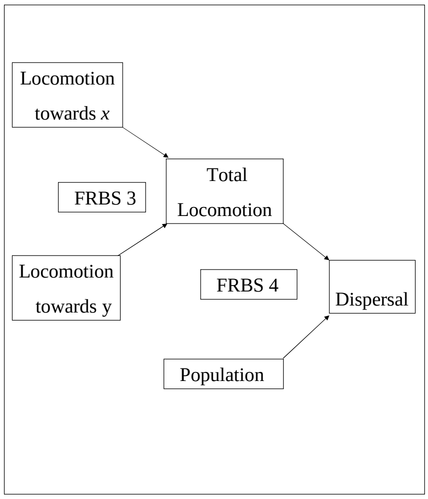

Figure 4: Fuzzy rule-based systems 3 and 4.

The initial condition for this problem is given by:

$$P(\mathbf{x}, 0) = P_0(\mathbf{x}), \quad \forall \mathbf{x} \in \Omega_1. \tag{6}$$

To determine the numerical solution for (1), along with the boundary conditions (2)-(5) and initial (6), the finite element method was used with a triangular uniform grid and linear functions. The Crank-Nicolson Method was employed for the time discretization.

3.1 Linguistic variables and rule base

Fuzzy sets are a way to represent imprecise information and knowledge. Velocity along the x is the output variable in the fuzzy rule-based system (FRBS 1) that depends on the variable of locomotion along the y. Velocity along the y is the output variable in the fuzzy rule-based system (FRBS 2) that depends on the variable of locomotion along the x, Figure 3. Velocities along the x (v_1) and y (v_2) are expressed by the term set {*very low*, *low*, *medium*, and *high*}. The velocity along the$x$ and y vary from 0.105 $m/hours$ to 0.17 $m/hours$ and from 0.04 $m/hours$ to 0.1 $m/hours$, in the classic model, and to satisfy the conditions of Peclet's Nucleus, it was observed that v_1 and v_2 should be less than 1.7 $m/hours$. The values of locomotion along the$x$ and y are expressed as {$constant$, $difficult$, $medium$, $easy$ } while the values of the population (P) are expressed in the term set {*very small*, *small*, *medium*, *large* and *very large*}. The membership functions that specify the meaning of the linguistic values are shown in Figure 5 and 6, for locomotion along the x (similar to locomotion along the y), velocity along the x (similar to velocity along the y), respectively. Thus, total locomotion, population and dispersal are represented by trapezoidal membership functions. The rule base that encodes the relationship between locomotion along the x and velocity along the y is summarized by:

- If locomotion along x is constant, then v_2 is constant.
- If locomotion along x is easy, then v_2 is medium.
- If locomotion along x is medium, then v_2 is high.
- If locomotion along x is difficult, then v_2 is low.

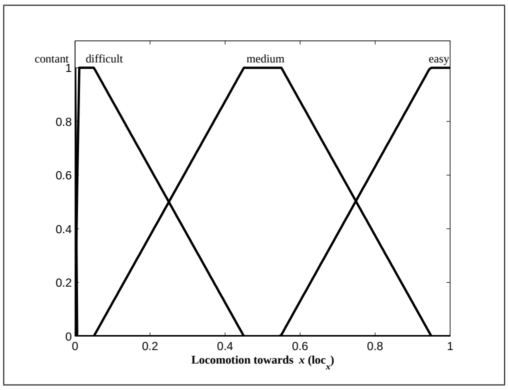

Figure 5: Membership functions for locomotion along the x direction (loc_x).

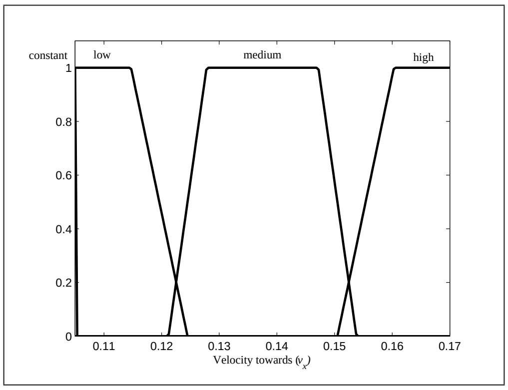

Figure 6: Membership functions for velocity along x (v_1).

The rule base of FRBS 2 is similar to FRBS 1. Total locomotion is the output variable in the fuzzy rule-based system (FRBS 3) that depends on the variables, locomotion along the x and locomotion along the y. Dispersal is the output variable in the fuzzy rule-based system (FRBS 4) that depends on the variables of total locomotion and population number, Figure 4. Population dispersal varies from $0.08m^2$/hours to $0.2m^2$/hours, since the results of the calculations of part of region A of Figure 1 are equal to 0.14 $m^2/hours$, which was considered the mean value of $medium$ dispersal in this model. Total locomotion is expressed by the term set $\{constant$, $very$ $difficult$, $difficult$, $medium$, and $easy\}$. Some examples of rules that encode the relationship between locomotion along x, locomotion along y and total locomotion are given by:

- If locomotion along x is constant and locomotion along y is constant, then total locomotion is constant.
- If locomotion along x is easy and locomotion along y is difficult, then total locomotion is medium.

The rule base that encodes the relationship between total locomotion, population and dispersal is defined analogously.

All the rule bases are processed using the Mamdani inference method with center of gravity defuzzification.

3.2 Fuzzy Computational Simulation

In order to define the type of ant locomotion as a function of the type of ground (e.g., rivers, hills, mountains and rocks) in the area of study to each node is allotted a value $[0,1]$ for movement in x and y according to its location in the area, see Figure 2.

The chosen trapezoidal membership function, which is associated to a fuzzy set A, $u_A(x)$, involving four parameters $[c_1\ c_2\ c_3\ c_4]$, see Figure 7, is given by:

$$u_A(x) = \begin{cases} \dfrac{x - c_1}{c_2 - c_1} & \text{if } c_1 \leq x < c_2 \\ 1 & \text{if } c_2 \leq x \leq c_3 \\ \dfrac{-x + c_4}{c_4 - c_3} & \text{if } c_3 < x \leq c_4 \end{cases} \tag{7}$$

If the triangular elements are located in the area of difficult locomotion, the value for locomotion in those elements is calculated by $c_1 + (c_4 - c_1) * rand$, where rand is equal to a random value in interval $(0, 1)$, $c_1 = 0$ e $c_4 = 0.45$. If the triangular elements are located in the area of medium locomotion, the value for locomotion in those elements is calculated by $c_1 + (c_4 - c_1) * rand$, where rand is equal to a random value of $(0, 1)$, $c_1 = 0.05$ and $c_4 = 0.95$.

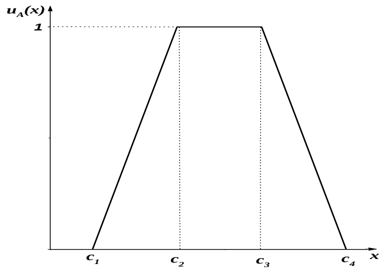

Figure 7: Parameters of trapezoidal membership functions.

If the triangular elements are located in the area of easy locomotion, the value for locomotion in those elements is calculated by $c_1 + (1 - c_1) * rand$, since the values of x that represent the degree of membership between 0 and 1 for easy locomotion vary from $c_1 = 0.548$ to 1. Thus, locomotion in direction x and y in each triangle is determined randomly depending on the domain of the function of pertinence corresponding to the classification of the region of Figure2. For horizontal and vertical locomotion, the triangular elements of the region of attraction are allotted the value of zero (0), which has a unitary fuzzy set. Therefore, each triangular element received a value for locomotion along the x and y.

To obtain the fuzzy output for each triangular element, for FRBS 2, we use the average of population values on each of the three nodes, assuming a linear approximation for α depending fuzzy-wise on the population on the element for $t = t_n + \frac{\Delta t}{2}$.

The value of velocity along x and velocity along y was found utilizing the FRBS 1 and FRBS 2 with the input variables values such as locomotion along y and locomotion along x, respectively, in each triangular element, see Figure 3. To determine the dispersal in each triangular element, the two FBRS were used: FRBS 3 and FRBS 4, Figure4. Using FRBS 3, the value for total locomotion in each triangle was determined based on the values of locomotion along the$x$ and y. The dispersal in the node of each triangle was determined based on the values of total locomotion and medium population, using FRBS 4.

Thus, x and y components of locomotion were determined only once for each triangle of region, prior to the first iteration, and total locomotion was determined based on the speed in directions x and y. The dispersion was calculated in each iteration for each triangle, since it depends on the total locomotion and on the population. The latter varies in each iteration.

To calculate population P on the boundaries τ_0 and τ_2, we determined the velocity along y depending on the locomotion in x for the triangles that belong to τ_0 and τ_2. To calculate population P on the boundaries τ_1 and τ_3 were calculated by determining the velocity along x, depending on the locomotion in y for the triangles that belong to τ_1 and τ_3. This also guarantees that $div(v) = \mathbf{0}$.

Figure 8 illustrates the initial condition, i.e., the *A. cephalotes* colony before it occupied the region of attraction. The graph in Figure 9 shows the population after of 33 hours, since ants take 33 hours to occupy the considered region of Figure 2.

4 Conclusion

This paper proposed the fuzzy model to study the occupation of the ant population in a given region of attraction. In the model, the ants occupied the region of attraction, as reported by [2] for a cassava monoculture. The main difference between the classic and fuzzy models (1) is that the fuzzy model exploits uncertainty parameters whereas the classic model does not. In the fuzzy model, dispersion is a fuzzy parameter that depends on the population and on the

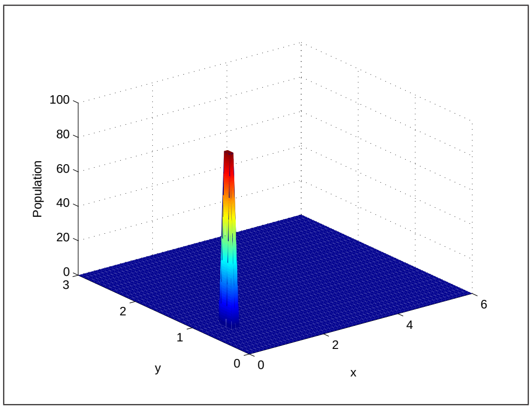

Figure 8: Initial population.

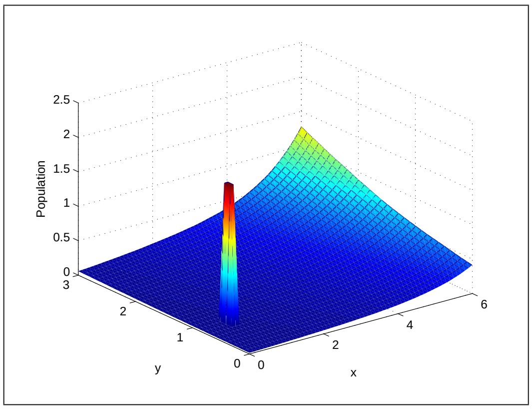

Figure 9: Population after 33 hours.

total locomotion, which is related to the facility or difficulty of locomotion that the ants will face depending on the terrain they traverse. In this model, the velocity along x and y is also a fuzzy parameter that depends on the locomotion along y and x, respectively. Thus, the authors believe that simulations made with the fuzzy model will more faithfully portray the phenomenon under study.

Acknowledgements

The authors thank FAPEMIG, the Research Foundation of the State of Minas Gerais.

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