

Composing a Portfolio in Stock Markets by using a Series Saturation Method

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***Abstract:** Here it is proposed a method for acting on stock markets which is similar to the traditional Markowitz one, but with a slightly modification introduced in the stock's returns series considered in the process. This modification, in fact an operation on the original series, was named here as saturation series method. The operation can be applied iteratively till to obtain the degree of saturation that one wants. Notice that the method has an invariant-under-operation limit series. After, it was done as experience a stock trade simulation by using the Markowitz strategy under several different degrees of saturation on the return series considered. A comparison among them shows that the consideration of saturated series in the game, in general increases the profits.*

Introduction

In the last years we had a huge diffusion of the Markowitz [5] strategy for stock markets. So many things were written about the fundamental concepts involved [6, 7], mainly on the concept of the risk measurement in a portfolio.

We are proposing here a concept of risk, by introducing a slightly difference with respect to the classical considerations from Markowitz. This modification, it consist in to apply an operation on the original series, and is named here as saturation series method. The operation can be applied iteratively till to obtain the degree of saturation that one wants. The method is done in a section below.

Notice that some suggestions for a slightly different risk definition it was done before in the literature (see the definitions of the average absolute deviation and of the semi-variance as defined in Ch.4.2 at [4], for instance).

The intuitive background for this approach actually is directed for keeping a psychological aspect in the stock markets environment. We are talking about the investors behavior that – in raw words – overestimate the impact when loosing, making it in general much more intense than the fairly winning situations ones.

The Markowitz strategy

The Markowitz strategy has as its stick holders, the following:

- It is necessary to diversify the stocks in a Portfolio (“do not put all the eggs...”) choosing the lowest correlated stocks to put together;
- It is necessary to balance the stocks in the Portfolio following the principle:
In the Markowitz plane (risk – the standard deviation mean × return – mean value) take the return and risk in the portfolio P as the composite form from the return and risk of the stocks that compose P. For instance, in the minimum possible case diversification – 2 stocks A, B composing P – with the proportion x, $P = x A + (1 - x) B$, the return and risk of P is:

$$\overline{R}_P = x \overline{R}_A + (1-x) \overline{R}_B \quad \text{and} \quad \sigma_P^2 = x^2 \sigma_A^2 + (1-x)^2 \sigma_B^2 + 2x(1-x) \sigma_{AB}$$

where σ_{AB} is the AB – covariation. The minimum risk in the composition of the stocks from the Portfolio is done using derivations for the value x_0 that satisfies:

$$2x \sigma_A^2 + 2(1-x) \sigma_B^2 + 2(1-2x) \sigma_{AB} = 0$$

or

$$x_0 = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$$

Moreover if ρ is the AB – correlation the curve composition of A, B in the Markowitz plane looks like the Figure 1.

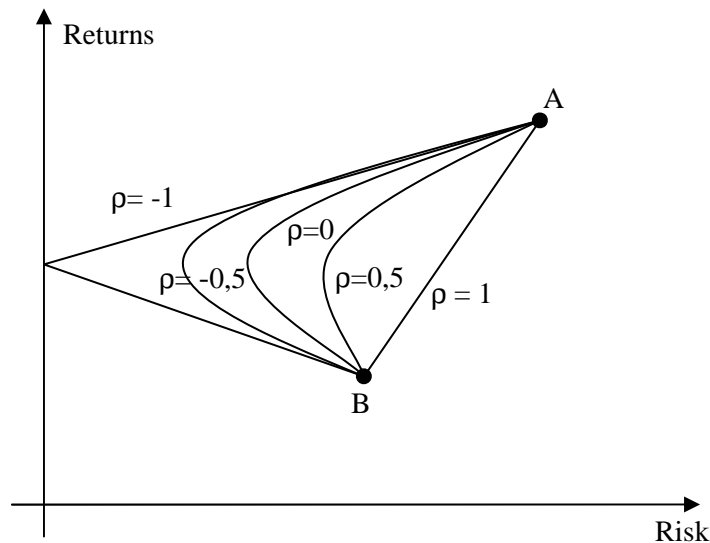


Figure 1: Risk x Return curves for composition of A and B.

Saturation Method for Series

Specifically: given the original series S for available stocks we will be introducing a modification in it until obtain a new “more defended” stable series in the sense that will be clear into the definition of the process itself.

Now, let us done in a inductive manner the process of the series S transform, building a sequence of series S_0, S_1, \dots , making

$$S_j : s_1^j, s_2^j, \dots, s_k^j, \dots$$

Define for this series the minimum square line

$$L_j : y = \alpha_j x + \beta_j$$

associated to the dispersion $\mathbf{N}^* \times S_j = \{ (x,y) : x = n \in \mathbf{N}^*; y = s_n^j \in S_j \}$ [2].

Get $S_0 = S$. If we have done the S_j series the S_{j+1} one will be the series with the elements:

$$s_k^{j+1} = s_k^j \quad \text{if} \quad s_k^j \leq \alpha_j k + \beta_j$$

$$s_k^{j+1} = \alpha_j k + \beta_j \quad \text{if} \quad s_k^j > \alpha_j k + \beta_j$$

Experimental Results

Let us make an experience performing a stock trade simulation by using the Markowitz strategy under several different degrees of saturation on the return stocks series considered. At the end we made a comparison among the different results obtained showing that the consideration of saturated series, in general increases the profits.

Our universe was the 10 bigger (in %) participants of the BOVESPA [1] - Portfolio namely:

bbdc4, bvmf3, csna3, ggbr4, itau4, petr3, petr4, usim5, vale3 and vale5.

We made the following choice as strategy:

Experiment 1: We used three periods for acting:

Period 1: From 05/01/2009 to 31/01/2009 (20 days), we choose the 2 lower correlated stocks (ggbr4 and bvmf3). Then following the Markowitz proportional calculus for the minimal possible risk we compose the Portfolio in the 21th day (39% - bvmf3 and 61% - ggbr4). The Portfolio was liquidated in the 30th day.

Period 2: From 20/01/2009 to 16/02/2009 (20 days), we choose the 2 lower correlated stocks (usim5 and petr3). Then following the Markowitz proportional calculus for the minimal possible risk we compose the Portfolio in the 31th day (28% - petr3 and 72% - usim5). The Portfolio was liquidated in the 40th day.

Period 3: From 03/02/2009 to 04/03/2009 (20 days), we choose the 2 lower correlated stocks (bvmf3 and bbdc4). Then following the Markowitz proportional calculus for the minimal possible risk we compose the Portfolio in the 41th day (83% - bvmf3 and 17% - bbdc4). The Portfolio was liquidated in the 50th day.

Experiment 2: We repeat the choice of the stocks and the periods as in the experiment 1 only making the Markowitz calculations according the first transform [the series (S_1)] of the stocks return series involved.

Experiments 3, 4 and 5: We repeat the choice of the stocks and the periods as in the experiment 1 only making the Markowitz calculations according the 3 transforms [the series (S_4 , S_{11} and S_{24} , respectively)] of the stocks return series involved.

Remark 1: We could re-calculate the composition for the 2-stocks Portfolio using the saturated series in each case with the sake of to choose lower correlations in our stocks universe. It was not done for comparison purposes.

Results of the Experiments

The experiments release the following results:

Experiment	Returns Period 1 (%)	Returns Period 2 (%)	Returns Period 3 (%)	Return Full Period (%)
1	9,60	-13,50	8,86	3,20
2	7,00	-13,27	8,84	1,00
3	7,00	-12,06	8,82	2,40
4	7,30	-9,60	8,87	5,60
5	7,30	-9,60	8,87	5,60

Table 1: Returns in each experiment.

Conclusions

In accordance with the conclusions in several another experiments made elsewhere by the authors [3, 8] we see in this case too, in the Table 1 that:

- The saturation method for series described above it works in critical negative situations (intuitively speaking because it has a defensive flavor and works better in critical situations, diminishing losses as seen in the column for period 2). Also, immediately after this critical event the method of saturation it works a little better than others (see the column for the period 3);
- The existence of an invariant-under-operation limit saturated series implies the saturation (invariance) of the returns in the system (as suggested by the rows 4 and 5 in the Table 1).
- Last, but not the least, we had more profit in the experiment 4 (or 5) when using the saturated series S_{11} other than in the case of the classical Markowitz strategy in experiment 1, with the original series S_0 .

Remark 2: We used the datae final day price in the next table for make the above experiments:

Data	bbdc4	bvmf3	csna3	ggbr4	itau4	petr3	petr4	usim5	vale3	vale5
05/01/2009	24,66	6,98	34,55	16,94	28,64	30,52	25,10	29,99	32,16	28,00
06/01/2009	25,46	7,12	36,59	17,64	29,81	31,02	25,45	30,55	33,05	28,95
07/01/2009	24,31	6,75	35,11	17,53	28,10	29,40	24,45	29,45	31,00	27,49
08/01/2009	24,35	6,78	37,55	18,73	28,27	30,95	25,50	30,67	32,38	28,51
09/01/2009	24,11	6,58	37,08	18,68	28,24	30,90	25,40	30,10	31,85	28,23
12/01/2009	23,23	6,40	34,48	17,03	27,04	28,64	23,75	28,86	29,31	26,18
13/01/2009	22,33	6,50	33,91	16,69	26,05	28,74	23,95	28,75	30,10	26,75
14/01/2009	20,98	6,35	33,21	15,95	24,09	27,68	23,30	27,48	28,70	25,70
15/01/2009	21,62	6,32	34,65	16,74	24,53	28,59	24,10	28,47	29,45	26,40
16/01/2009	21,17	6,35	35,50	17,02	23,69	29,00	24,28	29,34	30,00	26,60
19/01/2009	21,20	6,23	35,86	16,74	24,04	28,47	23,83	29,16	29,67	26,39
20/01/2009	20,08	5,80	33,47	15,95	22,39	27,45	23,06	27,74	28,36	25,30
21/01/2009	21,06	6,21	35,83	16,46	23,15	29,01	24,26	28,81	29,19	25,97
22/01/2009	20,87	6,18	35,50	15,79	22,50	28,31	23,64	28,93	29,23	25,85
23/01/2009	21,09	6,25	34,50	15,51	23,02	28,24	23,59	27,74	30,25	26,30
26/01/2009	21,12	6,43	34,99	15,65	22,87	28,59	23,80	26,93	30,82	26,95
27/01/2009	20,71	6,48	35,81	15,11	22,54	28,35	23,90	27,04	32,11	27,93
28/01/2009	21,67	6,85	37,07	15,31	24,21	30,45	25,20	28,50	33,94	29,37
29/01/2009	21,02	6,85	36,12	15,14	23,44	30,02	25,03	28,62	33,03	28,50
30/01/2009	20,77	6,69	35,18	14,98	23,13	30,37	25,03	28,29	32,50	28,01
02/02/2009	20,22	6,53	35,20	14,80	22,42	29,76	24,69	27,55	32,31	27,75
03/02/2009	20,29	6,66	36,34	15,54	22,75	30,91	25,58	27,75	33,91	29,00
04/02/2009	20,88	6,73	37,52	16,27	23,17	31,16	25,70	29,21	35,41	30,05
05/02/2009	21,58	6,93	38,75	16,58	24,16	31,50	26,00	30,28	36,95	31,35
06/02/2009	22,61	7,61	40,20	17,27	25,88	33,20	27,10	31,37	38,75	32,48
09/02/2009	22,65	7,20	39,08	16,79	25,48	33,61	27,39	29,94	37,60	31,52
10/02/2009	22,05	6,84	37,80	15,85	24,49	33,38	27,05	28,91	36,50	30,89
11/02/2009	22,42	6,62	36,78	15,87	24,44	33,00	26,90	28,10	35,48	30,29
12/02/2009	21,80	6,52	35,60	15,40	24,23	32,75	26,79	27,64	34,81	29,80
13/02/2009	22,53	6,96	37,11	15,87	25,24	33,92	27,60	28,52	36,39	30,69
16/02/2009	22,41	6,86	37,50	15,70	25,14	34,25	27,92	28,17	36,51	30,92
17/02/2009	21,51	6,51	35,86	14,85	23,65	32,31	26,40	26,90	34,11	29,06
18/02/2009	21,63	6,22	34,50	14,65	23,45	31,57	25,89	27,25	34,80	29,61
19/02/2009	21,38	6,10	34,20	14,34	23,24	32,20	26,30	27,21	35,02	29,79
20/02/2009	20,50	6,30	33,66	13,43	22,56	31,50	25,78	26,46	32,23	27,80
25/02/2009	20,98	6,05	32,01	13,06	22,32	32,25	26,25	25,69	30,50	26,51
26/02/2009	21,13	5,96	31,78	12,74	22,37	32,50	26,52	25,67	30,50	26,56
27/02/2009	20,68	5,95	31,35	12,62	21,97	32,70	26,40	25,41	30,87	26,83
02/03/2009	19,91	5,80	29,09	11,86	20,79	30,68	25,02	24,01	29,12	25,25
03/03/2009	19,81	5,99	30,50	12,02	20,69	30,50	24,80	24,11	29,68	25,80
04/03/2009	20,84	6,28	33,29	12,81	21,94	32,51	26,37	26,07	32,69	28,30
05/03/2009	20,40	6,13	31,20	12,00	21,78	32,10	26,00	24,46	31,22	27,20
06/03/2009	20,87	6,15	31,02	11,48	22,08	31,51	25,67	24,30	30,40	26,57
09/03/2009	20,27	6,19	30,70	11,31	21,82	31,59	25,70	23,27	29,70	25,77
10/03/2009	21,49	6,69	33,10	11,80	23,20	33,32	27,07	24,72	31,90	27,37
11/03/2009	21,30	6,72	32,90	11,90	23,25	33,53	27,21	24,60	32,22	27,48
12/03/2009	21,86	6,79	33,30	12,05	23,85	34,30	27,55	24,52	31,66	27,10
13/03/2009	21,70	6,75	32,34	11,67	24,00	34,40	27,70	24,50	30,82	26,60
16/03/2009	21,82	6,75	31,76	11,35	24,11	33,60	27,19	23,60	30,68	26,71
17/03/2009	22,20	6,87	31,90	11,20	24,64	34,60	27,98	23,30	31,45	27,05
18/03/2009	22,63	7,12	33,55	11,41	25,41	35,00	28,30	23,50	31,14	26,96
19/03/2009	21,91	6,79	34,10	11,45	24,76	36,32	29,25	25,00	31,80	27,30
20/03/2009	21,51	6,66	33,80	11,56	24,40	36,32	29,10	25,76	31,35	27,10

Table 2: The final period price for the stocks involved in the experiments.

Remark 3: In the experiment above we used the saturated series listed below in the next 6 tables.

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
06/01/2009	1,97	-1,53	-3,83	-5,75	-6,99
07/01/2009	-5,48	-5,48	-5,48	-5,62	-6,88
08/01/2009	0,44	-1,29	-3,61	-5,48	-6,77
09/01/2009	-3,04	-3,04	-3,51	-5,35	-6,66
12/01/2009	-2,81	-2,81	-3,40	-5,22	-6,55
13/01/2009	1,54	-0,93	-3,29	-5,09	-6,44
14/01/2009	-2,36	-2,36	-3,19	-4,96	-6,33
15/01/2009	-0,47	-0,69	-3,08	-4,82	-6,22
16/01/2009	0,47	-0,57	-2,97	-4,69	-6,11
19/01/2009	-1,93	-1,93	-2,87	-4,56	-6,00
20/01/2009	-7,41	-7,41	-7,41	-7,41	-7,41
21/01/2009	6,60	-0,21	-2,66	-4,30	-5,78
22/01/2009	-0,49	-0,49	-2,55	-4,16	-5,67
23/01/2009	1,12	0,03	-2,44	-4,03	-5,56
26/01/2009	2,80	0,15	-2,34	-3,90	-5,45
27/01/2009	0,77	0,27	-2,23	-3,77	-5,34
28/01/2009	5,40	0,39	-2,12	-3,64	-5,23
29/01/2009	0,00	0,00	-2,02	-3,50	-5,12
30/01/2009	-2,39	-2,39	-2,39	-3,37	-5,01
02/02/2009	-2,45	-2,45	-2,45	-3,24	-4,90

Table 3: The saturation series for bvmf3 (1st period).

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
20/01/2009	-3,72	-3,72	-3,72	-3,72	-3,72
21/01/2009	5,38	0,42	-2,22	-3,20	-3,50
22/01/2009	-2,47	-2,47	-2,47	-3,04	-3,33
23/01/2009	-0,25	-0,25	-1,96	-2,89	-3,17
26/01/2009	1,22	0,58	-1,82	-2,73	-3,01
27/01/2009	-0,85	-0,85	-1,69	-2,58	-2,85
28/01/2009	6,90	0,69	-1,56	-2,42	-2,68
29/01/2009	-1,43	-1,43	-1,43	-2,27	-2,52
30/01/2009	1,15	0,80	-1,29	-2,11	-2,36
02/02/2009	-2,05	-2,05	-2,05	-2,05	-2,20
03/02/2009	3,72	0,91	-1,02	-1,80	-2,03
04/02/2009	0,80	0,80	-0,89	-1,64	-1,87
05/02/2009	1,08	1,02	-0,76	-1,49	-1,71
06/02/2009	5,12	1,07	-0,62	-1,33	-1,55
09/02/2009	1,22	1,13	-0,49	-1,18	-1,38
10/02/2009	-0,69	-0,69	-0,69	-1,02	-1,22
11/02/2009	-1,15	-1,15	-1,15	-1,15	-1,15
12/02/2009	-0,76	-0,76	-0,76	-0,76	-0,89
13/02/2009	3,45	1,34	0,04	-0,55	-0,73
16/02/2009	0,96	0,96	0,18	-0,40	-0,57

Table 5: The saturation series for petr3.

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
06/01/2009	3,97	0,18	-4,44	-8,28	-10,62
07/01/2009	-0,63	-0,63	-4,35	-7,98	-10,19
08/01/2009	6,41	-0,02	-4,26	-7,67	-9,77
09/01/2009	-0,27	-0,27	-4,16	-7,37	-9,35
12/01/2009	-9,69	-9,69	-9,69	-9,69	-9,69
13/01/2009	-2,04	-2,04	-3,98	-6,76	-8,50
14/01/2009	-4,64	-4,64	-4,64	-6,46	-8,08
15/01/2009	4,72	-0,50	-3,80	-6,16	-7,66
16/01/2009	1,65	-0,60	-3,71	-5,86	-7,23
19/01/2009	-1,67	-1,67	-3,61	-5,55	-6,81
20/01/2009	-4,95	-4,95	-4,95	-5,25	-6,39
21/01/2009	3,10	-0,89	-3,43	-4,95	-5,97
22/01/2009	-4,24	-4,24	-4,24	-4,64	-5,54
23/01/2009	-1,81	-1,81	-3,25	-4,34	-5,12
26/01/2009	0,89	-1,18	-3,16	-4,04	-4,70
27/01/2009	-3,57	-3,57	-3,57	-3,74	-4,28
28/01/2009	1,31	-1,37	-2,97	-3,43	-3,85
29/01/2009	-1,12	-1,47	-2,88	-3,13	-3,43
30/01/2009	-1,07	-1,57	-2,79	-2,93	-3,01
02/02/2009	-1,22	-1,66	-2,70	-2,76	-2,76

Table 4: The saturation series for ggbr4.

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
20/01/2009	-5,12	-5,12	-5,12	-5,12	-5,12
21/01/2009	3,71	-0,37	-3,41	-4,65	-5,00
22/01/2009	0,41	-0,36	-3,34	-4,57	-4,96
23/01/2009	-4,29	-4,29	-4,29	-4,50	-4,91
26/01/2009	-3,01	-3,01	-3,21	-4,43	-4,87
27/01/2009	0,41	-0,30	-3,15	-4,36	-4,82
28/01/2009	5,12	-0,29	-3,08	-4,29	-4,77
29/01/2009	0,42	-0,27	-3,02	-4,22	-4,73
30/01/2009	-1,17	-1,17	-2,95	-4,14	-4,68
02/02/2009	-2,69	-2,69	-2,89	-4,07	-4,63
03/02/2009	0,72	-0,22	-2,82	-4,00	-4,59
04/02/2009	5,00	-0,20	-2,76	-3,93	-4,54
05/02/2009	3,53	-0,18	-2,69	-3,86	-4,49
06/02/2009	3,47	-0,16	-2,63	-3,78	-4,45
09/02/2009	-4,78	-4,78	-4,78	-4,78	-4,78
10/02/2009	-3,56	-3,56	-3,56	-3,64	-4,35
11/02/2009	-2,88	-2,88	-2,88	-3,57	-4,31
12/02/2009	-1,66	-1,66	-2,37	-3,50	-4,26
13/02/2009	3,09	-0,08	-2,30	-3,42	-4,21
16/02/2009	-1,24	-1,24	-2,24	-3,35	-4,17

Table 6: The saturation series for usim5.

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
03/02/2009	1,95	1,95	1,95	1,95	1,95
04/02/2009	1,04	1,04	1,04	1,04	1,04
05/02/2009	2,89	2,89	2,89	2,89	2,89
06/02/2009	8,94	3,81	3,81	3,81	3,81
09/02/2009	-5,69	-5,69	-5,69	-5,69	-5,69
10/02/2009	-5,26	-5,26	-5,26	-5,26	-5,26
11/02/2009	-3,32	-3,32	-3,32	-3,32	-3,32
12/02/2009	-1,53	-1,53	-1,53	-1,53	-1,53
13/02/2009	6,32	3,62	3,62	3,62	3,62
16/02/2009	-1,46	-1,46	-1,46	-1,46	-1,46
17/02/2009	-5,38	-5,38	-5,38	-5,38	-5,38
18/02/2009	-4,66	-4,66	-4,66	-4,66	-4,66
19/02/2009	-1,97	-1,97	-1,97	-1,97	-1,97
20/02/2009	3,17	3,17	3,17	3,17	3,17
25/02/2009	-4,13	-4,13	-4,13	-4,13	-4,13
26/02/2009	-1,51	-1,51	-1,51	-1,51	-1,51
27/02/2009	-0,17	-0,17	-0,17	-0,17	-0,17
02/03/2009	-2,59	-2,59	-2,59	-2,59	-2,59
03/03/2009	3,17	3,17	3,17	3,17	3,17
04/03/2009	4,62	3,20	3,20	3,20	3,20

Table 7: The saturation series for bvmf3 (3rd period).

Data	S ₀	S ₁	S ₄	S ₁₁	S ₂₄
03/02/2009	0,34	0,34	0,34	0,34	0,34
04/02/2009	2,83	1,24	1,24	1,24	1,24
05/02/2009	3,24	1,12	1,12	1,12	1,12
06/02/2009	4,56	1,00	1,00	1,00	1,00
09/02/2009	0,18	0,18	0,18	0,18	0,18
10/02/2009	-2,72	-2,72	-2,72	-2,72	-2,72
11/02/2009	1,65	0,65	0,65	0,65	0,65
12/02/2009	-2,84	-2,84	-2,84	-2,84	-2,84
13/02/2009	3,24	0,41	0,41	0,41	0,41
16/02/2009	-0,54	-0,54	-0,54	-0,54	-0,54
17/02/2009	-4,18	-4,18	-4,18	-4,18	-4,18
18/02/2009	0,55	0,05	0,05	0,05	0,05
19/02/2009	-1,17	-1,17	-1,17	-1,17	-1,17
20/02/2009	-4,29	-4,29	-4,29	-4,29	-4,29
25/02/2009	2,29	-0,30	-0,30	-0,30	-0,30
26/02/2009	0,71	-0,42	-0,42	-0,42	-0,42
27/02/2009	-2,18	-2,18	-2,18	-2,18	-2,18
02/03/2009	-3,87	-3,87	-3,87	-3,87	-3,87
03/03/2009	-0,50	-0,77	-0,77	-0,77	-0,77
04/03/2009	4,94	-0,89	-0,89	-0,89	-0,89

Table 8: The saturation series for bbdc4.

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