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Zero Double Hopf Reversible Bifurcation

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RESUMO

We say that a vector field X is reversible if there exists a linear involution R satisfying $RX = -XR$. An orbit solution γ of X is called symmetric if $R\gamma = \gamma$.

We also consider reversible systems X_λ of the form:

$$\dot{x} = X(x, \lambda) \tag{1}$$

where $X(Rx, \lambda) = -RX(x, \lambda)$ again, with $x \in \mathbb{R}^5$ and $\lambda \in (-\lambda_0, \lambda_0) \subset \mathbb{R}$ and with $X_\lambda(x)$ be a smooth parameter-dependent vector field.

Given an involution R, defined in $\mathbb{R}^5$, such that the fixed point set $Fix(R) = \{x \in \mathbb{R}^5 : R(x) = x\}$ has dimension 2, then we can choose a local coordinate system such that

$$R(x, y_1, z_1, y_2, z_2) = (-x, z_1, y_1, z_2, y_2).$$

It is easy to check that if the system (1) is R–reversible then $X(x, \lambda) = \lambda e_1 + Ax + \cdots$ where

$$A = \begin{pmatrix} 0 & & & & \\ & 0 & -\alpha_1 & & \\ & \alpha_1 & 0 & & \\ & & & 0 & -\alpha_2 \\ & & & \alpha_2 & 0 \end{pmatrix}. \tag{2}$$

Let Γ_λ be the space of all C^∞–germs of R–reversible vector fields X_λ in $(\mathbb{R}^5, 0)$ with $X_\lambda(0) = (\lambda, 0, 0, 0, 0)$ and $DX_\lambda(0) = A$ and $\lambda \in (-\lambda_0, \lambda_0)$. We endow Γ_λ with the C^∞–topology.

One of characteristic properties of reversible systems is that generically periodic orbits or invariant tori or minimal sets of such systems typically appear in one-parameter families. We consider generic codimension one local bifurcation in ODE's. We observe that we can have dimension of the center manifold greater than two in reversible systems due to persistent eigenvalues in the imaginary axis. We want to analyze the non wandering dynamics of this system. We can say here that the generic unfolding of this vector field is given by putting a parameter λ in the first equation.

Our first main result (Theorem 1) presents two different scenarios that we can have in a zero-Hopf-Hopf bifurcation: one of them is that we have a 1–parameter family of 2–dimensional invariant tori that shrinks and disappears after the bifurcation. The second case we have 1–parameter family of 2–dimensional invariant tori detached from the equilibrium point that attaches to it and after de bifurcation detaches from it again.

The second main result (Theorem 2) shows the existence of 4–dimensional invariant manifolds filled by heteroclinic orbits. Depending on the terms of order two of the vector field, this manifold can be compact(bounded) or not. When the bifurcation parameter goes to the bifurcation value then this invariant manifold shrinks and disappears.

Finally the third main result (Theorem 3) deals with the existence of critical points and families of periodic orbits. In the first case we have two 1–parameter families of periodic orbits, and when λ goes to the bifurcation value the two families shrink and disappear. In the second case we have one 1–parameter family of periodic orbits, and at the bifurcation the family shrinks, disappears and arises again.

Keywords: *periodic orbits, heteroclinic orbits, reversibility*

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