

## Calculus of variations on time scales with contingent epiderivatives

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### ABSTRACT

We propose the use of the contingent epiderivative as a tool of differentiation on the calculus of variations on time scales. By introducing the problem

$$\mathcal{L}(y) = \int_a^b L(t, y^*(t), D_{\uparrow}y(t)(u))d_{\uparrow}t \longrightarrow \min \quad (1)$$

$$y(a) = y_a, \quad y(b) = y_b,$$

we are able to unify the two approaches explored so far in the literature of time scales. Indeed, for  $u = 1$  problem (1) is reduced to the delta problem of the calculus of variations studied in [2, 3],

$$\int_a^b L(t, y^{\sigma}(t), y^{\Delta}(t))\Delta t \longrightarrow \min, \quad y(a) = y_a, \quad y(b) = y_b,$$

while for  $u = -1$  we get the nabla variational problem on time scales [1, 5]:

$$\int_a^b L(t, y^{\rho}(t), -y^{\nabla}(t))\nabla t \longrightarrow \min, \quad y(a) = y_a, \quad y(b) = y_b.$$

Moreover, studying (1) with  $u = \pm 1$  as a two-objective optimization problem we are able to extend the ideas of [4] and prove new necessary and sufficient Pareto optimality conditions on time scales. Some illustrative examples are given.

**Key words:** *contingent epiderivative, calculus of variations, time scales, Pareto optimality.*

## References

- [1] F. M. Atici, D. C. Biles and A. Lebedinsky, An application of time scales to economics, *Math. Comput. Modelling*, 43 (2006), no. 7-8, 718–726.
- [2] M. Bohner, Calculus of variations on time scales, *Dynam. Systems Appl.*, 13 (2004), 339–349.
- [3] R. A. C. Ferreira and D. F. M. Torres, Higher-order calculus of variations on time scales, in “Mathematical Control Theory and Finance”, pp. 149–159, Springer, 2008.
- [4] A. B. Malinowska and D. F. M. Torres, Necessary and sufficient conditions for local Pareto optimality on time scales, *J. Math. Sci. (N. Y.)*, in press.
- [5] N. Martins and D. F. M. Torres, Calculus of variations on time scales with nabla derivatives, *Nonlinear Anal.*, in press. DOI: 10.1016/j.na.2008.11.035

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