Exponential Stabilization of the Kirchhoff-Viscoelastic Equations

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ABSTRACT

The nonlinear wave equation

$$\partial_t^2 u + M(\|A^{1/2}u\|^2)Au = 0$$

was studied for several authors, see for example [1, 2, 3, 4, 5, 6, 12]; but until now the question about the global existence of solution for initial data taken in the usual Sobolev's spaces remains open. To obtain global solution to a class relative to the above equation, several authors [9, 10] to name but a few, have considered damping terms as A^2u, $A\partial_t u$, or $A^\alpha \partial_t u$ which gives strong estimates resulting in the convergence of the nonlinear term of the approximated solution.

In Nishihara [11] the author considers the wave equation with linear frictional damping and shows the existence of global solution for a class of large initial data in $D(A)$ spaces, non analytical but close to an analytical data. Nishihara's result is an important improvement about the question of existence of solution for the nonlinear Kirchhoff equation with weak dissipation, because it provides a large space where the initial data can be taken to produce strong existence result. In this paper we consider Kirchhoff-Viscoelastic Equations. The system in question is the following

$$\partial_t^2 u + (1 + \|A^{1/2}u\|^2)Au - \int_0^t g(t-\tau)Au(\tau)d\tau = 0, \tag{1}$$

$$u(0) = u_0, \qquad \partial_t u(0) = u_1 \tag{2}$$

where by H we are denoting separable Hilbert space. We show that for a class of initial data in $D(A)$, there exists global solutions for large data.

By A we are denoting an unbounded nonnegative self-adjoint operator satisfying

$$A\colon D(A) \subset H \to H, \quad \text{and}$$

[V1] The embedding $D(A^r) \hookrightarrow D(A^s)$ is compact for any $r > s \geq 0$. To prove the exponential decay of the solutions we use the following hypothesis on g:

[V2] The conditions,

$$0 < g(t) \in C^3, \qquad -\kappa g(t) \leq g'(t) \leq -cg(t)$$

$$|g''(t)| \leq Cg(t), \qquad \alpha =: 1 - \int_0^\infty g(\tau)\, d\tau > 0$$

For large initial data we show the global existence of solution for the Kirchhoff equation with memory, that is when $M(s) = 1 + s$, $M\colon \mathbb{R} \to \mathbb{R}$ and $N = 1$. Additionally, we show that the

corresponding solutions of the different Kirchhoff models we study in this paper, decay with the same rate as the relaxation function g.

Our strategy is to consider the existence of analytic solutions for any analytic initial data. Finally we show the existence of large solutions in the usual Sobolev's spaces $D(A)$ provided the initial data is close enough to an analytic initial data.

We summarized this result in the following theorem.

Principal Result. Let us denote by $(u_0, u_1) \in D(A) \times D(A^{1/2})$ such that

$$\|Au_0 - Av_0\|^2 + \|A^{1/2}u_1 - A^{1/2}v_1\|^2 \leq \epsilon$$

with v_0 and v_1 A-analytical data. Then the local solution u of (1)-(2) is globally defined and decay exponentially.

Key-Words: *Stabilization, Viscoelasticity, Existence, Hyperbolic Equations*

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