Existence of weak solution for Nonlocal Problem

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ABSTRACT

This work studies the existence of a class of nonlinear parabolic problems of nonlocal type with additive noise.

1 Introduction

Let D be a bounded, smooth open subset of $\mathbb{R}^n$, $n \geq 1$ and $T > 0$. We consider the following equation:

$$\begin{cases} u_t - a\left(\int_D u dx\right) \Delta u = \gamma u + f + \frac{\partial g}{\partial t} \;\; on \; Q = D \times]0, T[, \\ u(x, 0) = u_0(x) \; in \; D, u = 0 \; on \; \Sigma = \partial D \times]0, T[, \end{cases} \tag{1}$$

where $f \in L^2(0, T, L^2(D))$, $g \in C_0([0, T], H_0^1(D))$, $\gamma > 0$ constant, $u_0 \in H_0^1(D)$, ∂D a boundary of D and $a = a(s)$ a continuous function such that $0 < p \leq a(s) \leq P$ and with Lipschitz's constant L, where p and P are constants. These problems arise in various physical situations.

Remark 1

Firstly we will obtain an existence result of weak solution to system (1) with forcing terms $f + \frac{\partial}{\partial t} g$, where, as we said above, the functions g will not be assumed differentiable in a classical sense. This generalization covers a wide class of distributional forcing terms, and also the case when $\frac{\partial g}{\partial t}$ be a white noise. A possible interpretation of the forces $f + \frac{\partial g}{\partial t}$, is that it is composed of an *average* term f and a rapidly fluctuating part $\frac{\partial g}{\partial t}$.

Main result:

Theorem 1.1 *Let* $u_0 \in L^2(D)$, $f \in L^2(Q)$, $g \in C_0([0,T], H_0^1(D))$. *Then, there exists a solution* u *of (1), such that*

1. $u \in L^2(0,T; H_0^1(D)) \cap C([0,T]; L^2(D))$,
2. $u' - g' \in L^2(0,T; H^{-1}(D))$,
3. $u(0) = u_0$.

Keywords: *Nonlocal problem; parabolic; existence; additive noise.*

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