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Aspect algorithm for stabilization by output feedback, Lyapunov Equations, and Ricatti Equations for class descriptor system

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Abstract

A new approach to the design of output feedback controller is proposed, and the respective output feedback gains are obtained through the solution Riccati equation and Lyapunov equation using invariant subspaces. The objective is to reduce class descriptor system to a normal system, and gain from that change resolved the stabilization problem output feedback in the new system reduced using the Riccati Equation and Lyapunov Equations.
Keywords: Class Descriptor System, Stabilization, Riccati Equation, Lyapunov Equation.

1 Introduction

This paper deals the stabilization problem by static output feedback for class descriptor systems by using Riccati equations and Lyapunov Equations.

The class descriptor systems described by a set differential-algebraic equations in the form :

$$\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(t) \quad (1)$$

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (2)$$

where: $x_1 \in \Re^q$, $x_2 \in \Re^{n-q}$, $u \in \Re^m$ and $y \in \Re^p$; I_q is the identity matrix the finite size q and $J_4 \in \Re^{n-q \times n-q}$ is the invertible matrix. A new approach to the design of output feedback controller is proposed, and the respective output feedback gains are obtained through the solution Riccati equation and Lyapunov equation using invariant subspaces. The objective is to reduce class descriptor system to a normal system, and gain from that change resolved the stabilization problem output feedback in the new system reduced using the Riccati Equation and Lyapunov Equations.

The paper is organized as follows. The second section 2, presents the problem, introduces the basic concepts, the presented in [1], in section 3 is presented the problem output feedback using the Riccati and Lyapunov equations , is formulated an existence theorem for the output feedback matrix. Some conclusive commentaries finally are presented.

2 Preliminaries

The considered descriptor time-invariant systems are described by :

$$\tilde{E}\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) \quad (3)$$

$$y(t) = \tilde{C}x(t) \quad (4)$$

where: $x \in \mathcal{X} \sim \Re^n$, $u \in \mathcal{U} \sim \Re^m$, $y \in \mathcal{Y} \sim \Re^p$ and $\tilde{E} \in \Re^{n\times n}$, $rank\,(\tilde{E}) = q < n$; as the other matrices is an appropriate size with $rank\,(\tilde{B}) = m$ and $rank\,(\tilde{C}) = p$.

The basic problem to be solved is finding a control law for static output feedback for descriptor systems.

Thus considered the system

$$\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} J_1 - J_2J_4^{-1}J_3 & J_2 \\ 0 & J_4 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(t) \quad (5)$$

$$y(t) = \begin{bmatrix} C_1 - C_2J_4^{-1}J_3 & C_2 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \quad (6)$$

Thus, using $u(t) = Gy(t)$ for the system (5) , (6), obtained the equivalent representation the loop closed system for new coordinates change:

$$\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} J_1(G) & J_2(G) \\ 0 & J_4 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \quad (7)$$

where :

$$J_1(G) = (J_1 - J_2J_4^{-1}J_3) + B_1G(C_1 - C_2J_4^{-1}J_3)$$
$$J_2(G) = J_2 + B_1GC_2$$

3 Problem output feedback, Riccati Equations

In this section is considered the normal system, for resolved the class descriptor system. Given a system in the following state-space representation form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t);\ x(0) = x_0 \quad (8) \\ y(t) &= Cx(t) \quad (9) \end{aligned}$$

where the matrices $A = J_1 - J_2 J_4^{-1} J_3$, $B = B_1$ and $C = C_1 - C_2 J_4^{-1} J_3$.

Define the functional $J(G)$ as follows:

$$J(G) = \frac{1}{2} \int_0^T [y'(t)Qy(t) + u'(t)Ru(t)]dt \quad (10)$$

where $u(t) = Gy(t)$ and $\sigma(A+BGC) \in C^-$; Assuming that the matrices Q and R are symmetric and R is positive definite with the positive semi-definite state weighting matrix Q and the positive definite control weighting matrix R.

Furthermore, the next algebraic Riccati equation will play an important role in the analysis below:

$$A'P + PA - PBR^{-1}B'P + C'QC = 0 \quad (11)$$

The next step is then to verify whether the set of linear equations

$$R^{-1}B'P = -GC = F \quad (12)$$

has a solution G in [4], [3].

3.1 Aspect algorithm the Output Feedback matrix

The theorem (3.1) below relates the concept of (C, A, B)-invariant subspaces for the existence of a output feedback control law $u(t) = Gy(t)$ that S-stabilizes the closed loop system.

Theorem 3.1 *Considered the normal system (8) , (9) with (A, B), stabilizable (C, A) detectable, with C full row rank. There exists an output feedback matrix $G : \mathcal{Y} \longrightarrow \mathcal{U}$, with $F = -GC$ such that $\sigma(A+BGC) \in \mathcal{C}^-$, if and only if the following conditions are verified for exist the matrices P and G such that P solves (11) with $Q = Q'$ and $R^{-1}B'P = -GC = F$.* ⋄

Proof:
The proof is similar to the proof of Theorem in [1], [2].
The following basic procedure is proposed to compute a stabilizing static output feedback when $m + p > q$.

Step 1 :
Considered the matrices $A = J_1 - J_2 J_4^{-1} J_3$, $B = B_1$ and $C = C_1 - C_2 J_4^{-1} J_3$ in the new normal system.

1.1) Find the decomposition $C \begin{bmatrix} M_1 & M_2 \end{bmatrix} = \begin{bmatrix} \bar{C}_1 & 0 \end{bmatrix}$ and calculate the matrices A_{12}, A_{22}:

$$\begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} = \begin{bmatrix} \bar{M}_1' \\ \bar{M}_2' \end{bmatrix} AM_2$$

where

$$\begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} \bar{M}_1' \\ \bar{M}_2' \end{bmatrix} = I_n;$$

1.2) Solve the Lyapunov equation where P_1 is the stabilizing solution of the Riccati equation

$$A_{11}'P_1 + P_1 A_{11} - P_1 B_1 R^{-1} B_1' P_1 + C_1' Q C_1 = 0 \quad (13)$$

Step 2 : Then $minJ(G)$ exists for all y_0 and is attained by $G^* = G$. Moreover, $J(G^*) = y_0' P y_0$. and P solves (11).
Step 3 : Calculate the output feedback matrix that stabilizes the closed loop system, as the only solution of $\sigma(A - BR^{-1}B'PC) \in \mathcal{C}^{-1}$ and $R^{-1}B'PC = -BGC$.

4 Conclusion

The main contribution was to present an adaptation of the characterization of the quadratic Lyapunov equations to solve the problem of stabilization for a class of descriptor systems with the aid of Riccati equations. Thus in this paper, the problem of output feedback stabilization of linear systems was a new approach to the design of output feedback controller by using Riccati equation.

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