Approach for stabilization by output feedback, invariant subspace, Lyapunov Equations, and Ricatti Equations

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ABSTRACT

The algebraic Riccati equation is either of the following matrix equations:
the continuous time algebraic Riccati equation (CARE):

$$A'P + PA - PBB'P + Q = 0 \tag{1}$$

The use notion invariant subspaces (C, A, B) can be used as an intermediary for the construction of the feedback output. Thus, the paper presented anew approach to the design of output feedback controller is proposed, and the respective output feedback gains are obtained through the solution of a Riccati equation and Lyapunov equation using invariant subspaces.

The considered linear time-invariant system are described by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

$$y(t) = Cx(t) \tag{3}$$

where: $x \in \mathcal{X} \sim \Re^n$, $u \in \mathcal{U} \sim \Re^m$, $y \in \mathcal{Y} \sim \Re^p$. It is also assumed that B is full column-rank , C is full row-rank and that (C, A, B) is stabilizable and detectable. The studied problem is to find a static output feedback control law

$$u(t) = Gy(t) \quad , \quad G \in \Re^{m \times p} \tag{4}$$

such that $\sigma(A + BGC) \in \mathcal{C}^-$, or equivalently, the closed-loop system is asymptotically stable.

As in [4], some concepts and definitions are used geometric control theory [5]. It is know that an subspace $\mathcal{V} \subset \mathcal{X}$ is (A, B)-invariant if there exists $F : \mathcal{X} \longrightarrow \mathcal{Y}$ such that $(A + BF)\mathcal{V} \subset \mathcal{V}$, or equivalently , $A\mathcal{V} \subset \mathcal{V} + Im\,B$. For dually an subspace $\mathcal{T} \subset \mathcal{X}$ is (C, A)-invariant if exist $L : \mathcal{Y} \longrightarrow \mathcal{X}$ such that $(A + LC)\mathcal{T} \subset \mathcal{T}$, or equivalently, $\mathcal{T} \supset A(\mathcal{T} \cap Ker\,(C))$.

Given the system in the following state-space representation form:

$$\dot{x}(t) = Ax(t) + Bu(t);\ x(0) = x_0 \tag{5}$$

$$y(t) = Cx(t) \tag{6}$$

The next algebraic Riccati equation will play an important role in the analysis below:

$$A'P + PA - PBR^{-1}B'P + C'QC = 0 \tag{7}$$

Then the solution the Riccati equation use the following result in [6]:

Theorem 0.1 *[6] Assume that there exist P and G such that $X = P$ solves (7) with and $R^{-1}B'P = -GC = F$. Then exists for all y_0 and is attained by G. Moreover, conversely, if exists for all x_0, then there exist matrices X and G such that P, X satisfies (7), $\sigma(A - BR^{-1}B'X) \in \mathcal{C}^{-1}$ and $R^{-1}B'P = -GC = F$.*

From Theorem ([6]) that in case the optimization problem has a solution the optimal feedback is unique and given by $G = -R^{-1}B'XC'(CC')^{-1}$.

In The next step is then to verify whether the set linear equations

$$R^{-1}B'P = -GC = F \tag{8}$$

has a solution G.

The theorem (0.2) below relates the concept of (C, A, B)-invariant subspaces for the existence a output feedback control law $u(t) = Gy(t)$ that S-stabilizes the closed loop system.

Theorem 0.2 *Considered the system (2) , (3) with* (A, B)*, stabilizable* (C, A) *detectable, with* C *full row rank. There exists an output feedback matrix* $G : \mathcal{Y} \longrightarrow \mathcal{U}$*, with* $F = -GC$ *such that* $\sigma(A + BGC) \in \mathcal{C}^-$*, if and only if the following conditions are verified for some exist* $P = P'$ *and* G *such that* $X = P$ *solves (7) with* $Q = Q'$*.* $\diamond$

Proof:
Similarly the proof of Theorem in [2], [1].

The following basic procedure is proposed to compute a stabilizing static output feedback when $m + p > n$.

Step 1 :
1.1) Find the decomposition $C\begin{bmatrix} M_1 & M_2 \end{bmatrix} = \begin{bmatrix} \bar{C}_1 & 0 \end{bmatrix}$ and calculate the matrices A_{12}, A_{22}.
1.2) Solve the Lyapunov equation where P_1 is the stabilizing solution of the Riccati equation

$$A'_{11}P_1 + P_1A_{11} - P_1B_1R^{-1}B'_1P_1 + C'_1QC_1 = 0 \tag{9}$$

Step 2 : Then $minJ(G)$ exists for all y_0 and is attained by $G^* = G$. Moreover, $J(G^*) = y'_0Py_0$. and P solves (7).
Step 3 : Calculate the output feedback matrix that stabilizes the closed loop system, as the only solution of $\sigma(A - BR^{-1}B'PC) \in \mathcal{C}^{-1}$ and $R^{-1}B'PC = -BGC$.

Conclusion

Some base results had been presented based in concept of (C, A, B) invariant subspace e its relation with the solution of stabilization problem using output feedback , by Lyapunov equations. Thus in this paper, the output feedback stabilization problem in linear systems was a new approach to the design of output feedback controller, by using the equations Riccati.

Key-words: *Stabilization, Equations Lyapunov and Riccati, Invariant Subspace.*

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