Preinvex Functions in Vector Continuous-Time Optimization Problems

Valeriano Antunes de Oliveira
Universidade Federal de Uberlândia - UFU
Faculdade de Ciências Integradas do Pontal - FACIP
E-mail: valeriano@pontal.ufu.br

ABSTRACT

Let X be a nonempty subset of the Banach space $L_\infty^n[0,T]$, ϕ be a map from X into $\mathbb{R}^p$ and $f(x(t),t) = \xi(x)(t)$, $g(x(t),t) = \gamma(x)(t)$, where $\xi : X \to \Lambda_1^p[0,T]$ and $\gamma : X \to \Lambda_1^m[0,T]$, be given applications. $L_\infty^n[0,T]$ denotes the space of all n-dimensional vector valued Lebesgue measurable functions, which are essentially bounded, defined on the compact interval $[0,T] \subset \mathbb{R}$, with norm $\|\cdot\|_\infty$ defined by

$$\|x\|_\infty = \max_{1\le j\le n} \operatorname{ess\,sup}\{|x_j(t)|,\ 0 \le t \le T\},$$

where for each $t \in [0,T]$, $x_j(t)$ is j-th component of $x(t) \in \mathbb{R}^n$ and $\Lambda_1^m[0,T]$ denotes the space of all m-dimensional vector-valued functions which are essentially bounded and Lebesgue measurable, defined on $[0,T]$, with the norm $\|\cdot\|_1$ defined by

$$\|y\|_1 = \max_{1\le j\le m} \int_0^T |y_j(t)|dt.$$

The following vector continuous-time optimization problem is considered along with its scalar Lagrangean-type dual:

$$\begin{array}{ll} \text{Minimize} & \phi(x) = \displaystyle\int_0^T f(x(t),t)dt \\ \text{subject to} & g(x(t),t) \le 0 \text{ a.e. in } [0,T], \\ & x \in X; \end{array} \qquad \text{(VCP)}$$

$$\begin{array}{ll} \text{Maximize} & \psi(v,u) \\ \text{subject to} & u(t) \ge 0 \text{ a.e. in } [0,T], \end{array} \qquad \text{(SDP)}$$

where $\psi : V \times L_\infty^m[0,T] \to \mathbb{R}$ is given by

$$\psi(v,u) = \inf_{x\in X} \int_0^T [v'f(x(t),t) + u'(t)g(x(t),t)]dt$$

and $V = \{v \in \mathbb{R}^p : v_1 + \cdots + v_p = 1 \text{ and } v > 0\}$.

This kind of problems were studied, for example, in Mishra et al. [4], Nobakhtian and Pouryayevali [5, 6], de Oliveira and Rojas-Medar [7] and Zalmai [10].

Here, a nonconvex Gordan Transposition Theorem (see Mangasarian [3]) is developed in the context of continuous-time programming. Then optimality conditions of saddle point-type are obtained and the Geoffrion scheme (see Geoffrion [1]) is applied for (VCP) in the nondifferentiable case. Further, duality theorems are furnished regarding the scalar dual problem (SDP). It is well known that convexity is always used on establishing the cited results. In this work the concept of preinvexity is used instead.

Scalar duality for vectorial programming problems were first investigated by Osuna-Gómez et al. in [8].

Preinvex functions were introduced in Hanson and Mond [2] as a generalization of non-differentiable invex functions. Afterwards, Weir and Mond [9] worked with preinvex functions in multiple objective problems.

References

[1] A.M. Geoffrion, Proper efficiency and the theory of vector maximization, *J. Math. Anal. Appl.*, 22 (1968) 618–630.

[2] M.A. Hanson, B. Mond, Convex transformable programming problems and invexity, *J. Inform. Optim. Sci.*, 8 (1987) 201–207.

[3] O.L. Mangasarian, "Nonlinear Programming", SIAM Classics in Appl. Math., vol. 10, SIAM, Philadelphia, 1994.

[4] S.K. Mishra, S.Y. Wang, K.K. Lai, Nonsmooth continuous-time multiobjective optimization problems with invexity, *WSEAS Transactions on Mathematics*, 5 (2006) 635–639.

[5] S. Nobakhtian, M.R. Pouryayevali, Optimality criteria for nonsmooth continuous-time problems of multiobjective optimization, *J. Optim. Theory Appl.*, 136 (2008) 69–76.

[6] S. Nobakhtian, M.R. Pouryayevali, Duality for nonsmooth continuous-time problems of vector optimization, *J. Optim. Theory Appl.*, 136 (2008) 77–85.

[7] V.A. de Oliveira, M.A. Rojas-Medar, Continuous-time multiobjective optimization problems via invexity, *Abstr. Appl. Anal.*, 2007, Article ID 61296, 11 pp.

[8] R. Osuna-Gómez, A. Rufián-Lizana, P. Ruíz-Canales, Duality in nondifferentiable vector programming, *J. Math. Anal. Appl.*, 259 (2001) 462–475.

[9] T. Weir, B. Mond, Preinvex functions in multiple objective optimization, *J. Math. Anal. Appl.*, 136 (1988) 29–38.

[10] G.J. Zalmai, Proper efficiency conditions and duality models for a class of nonsmooth continuous-time multiobjective fractional programming problems, *Southeast Asian Bull. Math.*, 27 (2003) 155–186.