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Proper Pareto optimality for nonconvex and nonsmooth multicriteria optimization problems

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ABSTRACT

Multicriteria optimization problems have been introduced and studied in various ways. In this work, we will consider such problems whose ideal situation is determining "simultaneous minimum" of a set of functions $f_i,\ i=1,...,m$ over some domain $X \subseteq \mathbb{R}^n$. In general, may not exist a point in X such that all f_i attain their minimum and another concept of solution is Pareto optimality.

Consider a nonempty subset $X \subseteq \mathbb{R}^n$ and a set of functions $f_i : X \to \mathbb{R}$, $i=1,..,m.$ We will consider the problem

$$\begin{aligned}&\text{Minimize } (f_1(x),...,f_m(x))\\&\text{subject to } x\in X.\end{aligned} \qquad \textbf{(P)}$$

Recall that $x^* \in X$ is said to be Pareto optimal for (P) if there is no $x\in X$ such that $f_i(x) \le f_i(x^*),\ i=1,...,m$ with strict inequality for some i.

Many modifications of this concept have been studied. In this work, we will focus on the proper Pareto optimality, in the sense of Geoffrion [2]: An element $x^* \in X$ is said to be properly Pareto optimal for (P) if one can find a constant $d>0$ such that for all i, the system of inequalities

$$\begin{cases} f_i(x) < f_i(x^*) \\ f_i(x)+df_j(x) < f_i(x^*)+df_j(x^*),\ j\neq i \end{cases}$$

has no solution $x\in X$.

This concept of solution is very interesting because, given any criterion, the marginal loss in that criterion relative to a gain in some other criterion is bounded from above.

The set $P(X)$ of Pareto optimal solutions of (P) may have quite intricate structure. We will be interested in the case where *all* points of the domain are Pareto optimal. Of course, it can be achieved by cheating when we restrict the domain X to its subset $Y=P(X)$. In this case, we will obtain $P(Y)=Y$. Our objective is to identify some situations in which $P(X)=X$ for "non trivial" domains X.

Very recently, Šipošová [4] have been established a range of sufficient conditions for proper Pareto optimality of all points in nontrivial domains of multicriteria problems, whose functions are continuously differentiable and convex.

In this work, we extend the results obtained by Šipošová [4] for some nonsmooth and nonconvex problems. To do this, we will consider Lipschitz continuous functions and we use the results of nonsmooth analysis proposed by Clarke [1]. Also, we will suppose that the functions of (P) are invex. There is many notions of invexity and we will use those introduced by Phuong, Sach and Yen [3].

Keywords *: Invex functions, multiobjective problems, Pareto optimality, Nonsmooth analysis.*

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