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Numerical schemes for internal waves interacting with topography

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ABSTRACT

In the context of ocean dynamics, a reduced strongly nonlinear one-dimensional model for the evolution of internal waves over an arbitrary seabottom with submerged structures was derived in [12, 13]. This model is a generalization of the one proposed in [4]. The reduced model aims at obtaining an efficient numerical method for a two-dimensional problem with two layers containing inviscid, immiscible, incompressible and irrotational fluids of different densities. The upper layer is shallow compared with the characteristic wavelength at the interface of the two-fluid system, while the bottom region's depth is comparable to the characteristic wavelength. The non-linear evolution equations describe the behaviour of η (the internal wave elevation at the interface) and u (the mean upper-velocity) for this water configuration. We intend to use this strongly nonlinear model to study the interaction of large amplitude internal waves with multiscale topography profiles. The dynamics include wave scattering, dispersion and attenuation among other phenomena. The refocusing and stabilization of solitary waves for the large levels of nonlinearity allowed by this kind of models is the goal of current research.

To solve our model approximately we considered a numerical scheme based on the method of lines. Two particularities of the model deserve special attention: the dispersive term with a singular integral operator and the variable coefficient accounting for the topography information, which could be in a rapid scale. As used with success in [10], to deal numerically with a dispersive term an auxiliary variable V is introduced. The evolution in time is made on η and V. We implemented a fourth order Runge-Kutta time-integration scheme, which was also the choice in [4], and a fourth order approximation using a centered five point formula for the spatial derivative. Compared with some predictor-corrector schemes and spectral spatial derivatives, respectively, this is our best choice in terms of stability, up to now. When the nonlinearity is weak and the bottom is flat, we recover u from η and V (after each time step) by going to Fourier space via an FFT . In the presence of rough bottom (variable coefficients) or strong nonlinearity, this efficient strategy does not work and we must solve a linear algebraic system involving spectral matrices. By using a spectral matrix instead of an FFT, we are only paying a price in complexity but not in accuracy. This is an ongoing research. The spectral approach may not be the best for multiple scale problems, and we are currently investigating this. Some preliminary results from the Matlab implementations will be shown, including periodic topography experiments and solitary waves solutions.

The research is relevant in oil recovery in deep ocean waters, where salt concentration and differences in temperature generate stratification in such a way that internal waves can affect offshore operations and submerged structures. Another example of application, in the context of atmosphere dynamics, is the effect on the topographic form drag which is of importance in the study of pollution dispersion in an urban area.
Keywords: *Internal waves, Inhomogeneous media, Asymptotic theory*

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