ISSN 1984-820X
Anais do CNMAC
v.3
PUBLICAÇÃO DA SBMAC
CNMAC2010
XXXIII CONGRESSO NACIONAL DE MATEMÁTICA APLICADA E COMPUTACIONAL

# Nonlinear Thermoelastic Isotropic Rock Deformation with Radial Symmetry.

**Artur Lewis Gower** *
Instituto de Matemática Aplicada, Estatiśtica e Computação Científica, IMECC, Unicamp,
CEP 13083-859, Campinas, SP
E-mail: arturgower@gmail.com

## ABSTRACT

The objective of this research project is to determine a nonlinear model for thermoelastic rock deformation, then verify if the nonlinearity considerably affects the amplitude dispersion and arrival times of a source such as an explosion of dynamite.

To simplify the problem the model will be built with radial symmetry. In this way I can maintain better error bounds and still conclude if the nonlinearity considerably alters the results in relation to the linear elastic wave equation. The numerical simulations will focus on maintaining high resolution [1].

The foundations of this project will be in differential geometry, hence any coordinate system can be employed and I can explore a covariant treatment of the theory. For example, demanding that the balance of energy be covariant, results in, as a consequence; the balance of linear and angular momentum; and in the first step of giving a functional form to the stress tensor [2]. That is

$$\sigma^{ab}(x,t) = \rho(x,t)\frac{\partial e(x,t,g)}{\partial g_{ab}} \text{ (In spatial coordinates)}$$

where $\sigma$ is the Cauchy stress tensor, $\mathbf{g}$ is the Riemannian metric, and $\phi(X,t) = x$ is the position of the "particle" at time t, which originated at $X$.

The fundamental equations used will be the following:

$$\rho_{ref}(X) = J(x,t)\rho(x,t) \text{ (conservation of mass);} \tag{1}$$

$$\rho_{ref}\frac{\partial \mathbf{V}}{\partial t} = \mathbf{DIV\ P} \text{ (balance of momentum);} \tag{2}$$

$$\mathbf{S} = \mathbf{S^T} \text{ (balance of angular momentum);} \tag{3}$$

$$\rho_{ref}\frac{\partial E}{\partial t} + \mathbf{DIV\ Q} = \mathbf{DIV\ P} \text{ (balance of energy);} \tag{4}$$

$$\rho_{ref}N\frac{\partial \Theta}{\partial t} + \frac{\partial \Psi}{\partial t} - \mathbf{S} : \mathbf{D} + \frac{1}{\Theta}\langle \mathbf{Q}, \mathbf{GRAD}\ \Theta\rangle \leq 0 \text{ (reduced dissipation inequality);} \tag{5}$$

$$E = \Psi + N\Theta \text{ (relation between internal energy and free energy).} \tag{6}$$

Where, in order of appearance: $\rho$, $\rho_{ref}$ are density and initial (material) density; $\mathbf{V} = \partial_t\phi$; $\mathbf{P}$, $\mathbf{S}$ are the first and second Piola-Kirchhoff stress tensor; $\mathbf{Q}$ is the piola transform of the heat flux; $\mathbf{D} = \phi^*(\mathcal{L}_v\mathbf{g})$, the pull-back of the Lie derivative of the metric; $\Theta$, $N$ are temperature and entropy; $E$ is the internal energy.

*bolsista de Mestrado CNPq

ISSN 1984-820X
Anais do CNMAC
v.3
PUBLICAÇÃO DA SBMAC
CNMAC2010
XXXIII CONGRESSO NACIONAL DE MATEMÁTICA APLICADA E COMPUTACIONAL

These equations are formally ill-posed until the nature of the material is determined [3]. One way of doing this is to specify $\mathbf{S}, \mathbf{Q}, N$ and $\Psi$ as functions of $\phi$, the motion of the body, and the temperature $\Theta$. In this project these functional forms will be relevant for thermoelasticity.

At the present moment I have no numerical results. Although shorty I will begin numerical simulations; the results, and an analysis of them, will be included in the poster. The project is still in the theoretical phase, i.e. I am in the process of justifying the choice of the free energy function, which in our thermoelastic context will be a function of the invariants of $C$ and $\Theta$, the Cauchy-Green tensor and temperature. Hence the stress tensor will depend on the same parameters and also on the Riemannian metric. Then what remains is to specify the heat flux, which we approximate as a linear function of $\nabla\Theta$ (the temperature gradient).

**Key Words:** *Nonlinear Elasticity, Thermoelasticity, Isotropic Elastic Solids*

## References

[1] *Randall J. LeVeque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2004.*

[2] *J.E. Marsden, T.J. Hughes, Mathematical Foundations of Elasticity, Dover, 1994.*

[3] *C.Truesdell, W. Noll, The Non-Linear Field Theories of Mechanics, third edition, Springer-Verlag, 2004.*

[4] *G. B. Whitham, Linear and nonlinear waves , Wiley, New York, 1974.*